Introduction to laser plasma interaction (in the moderate intensity regime)

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Outline

- Laser-plasma interaction regimes
  - (not-too intense) electromagnetic waves in an unmagnetized plasma: dispersion relation, propagation, collisional absorption
- Laser-solid interaction; laser-driven ablation: mechanism, scaling laws for absorption, ablation pressure, mass ablation rate
- Laser-driven acceleration; rocket model; hydrodynamic efficiency
- Rayleigh-Taylor instability of laser-accelerated thin targets
Literature

Preface:

why great interest in

laser-matter interaction
Pulsed laser – solid interaction

can generate extreme states of matter
(temperature, density, pressure, e.m. fields)

can be used to drive inertial confinement fusion targets
Which lasers?

<table>
<thead>
<tr>
<th></th>
<th>long pulse (ns)</th>
<th>Short pulse (sub ps), Table-top</th>
<th>Short pulse Performance</th>
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</thead>
<tbody>
<tr>
<td>$E$</td>
<td>$0.1 – 2000 \text{ kJ}$</td>
<td>$\leq 1 \text{ J}$</td>
<td>$\leq 1 \text{ kJ}$</td>
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<tr>
<td>$Dt$</td>
<td>$1 – 10 \text{ ns}$</td>
<td>$10 \text{ fs} – 1 \text{ ps}$</td>
<td>$\leq 1 \text{ ps}$</td>
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<td>$W$</td>
<td>$0.1 – 500 \text{ TW}$</td>
<td>$&lt; 10 \text{ TW}$</td>
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<td>$r_f$</td>
<td>$0.1 – 1 \text{ mm}$</td>
<td>$\leq 10 \mu\text{m}$</td>
<td>$\sim 10 \mu\text{m}$</td>
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<tr>
<td>$I$</td>
<td>$\leq 10^{15} \text{ W/cm}^2$</td>
<td>$10^{17} – 10^{19} \text{ W/cm}^2$</td>
<td>$\leq 10^{22} \text{ W/cm}^2$</td>
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In all cases, wavelength $\lambda = 0.35 – 1.5 \mu\text{m}$
Collisional and non collisional & relativistic regimes of Laser - plasma interaction

Laser $\rightarrow$ accelerated electrons $\rightarrow$ collisions with electrons $\rightarrow$ collisions with ions

Three regimes of interaction:

- Relativistic, kinetic effects $\Rightarrow$ Mulser’s lectures
- Kinetic effects (distorted distribution function; parametric, instabilities) $\Rightarrow$ Tikhonchuk’s lectures
- Collisional absorption in a thermal plasma \((inverse\ bremsstrahlung)\Rightarrow these\ lectures\)

(Approximately) characterized by the value taken by a single dimensionless parameter

\[ a_e^2 = \left( \frac{v_{osc}}{c} \right)^2 = 7.3 \times 10^{-5} I_{ld} \lambda_{um}^2 \]
Laser pulses can generate extreme states of matter, e.g., very high pressures.

\[ p \sim (2/3) \varepsilon \]

\[ \varepsilon \sim \frac{E}{L^3} \]

\[ \Delta t \sim \text{ps} \Rightarrow L \sim r_f \sim 10 \mu m \] [no hydrodynamics, no conduction]

\[ \varepsilon \sim \frac{E}{L^3} \sim 10^9 E_j \text{ J/cm}^3 \Rightarrow p \sim 10 E_j \text{ Gbar} \]

\[ \Delta t \sim \text{ns} \Rightarrow L \sim c_s \Delta t \sim 300 \text{ } t_{\text{ns}} \mu m \] [hydrodynamics]

\[ \varepsilon \sim \frac{E}{L^3} \sim 4 \times 10^7 E_{kj} (t_{ns})^{-3} \text{ J/cm}^3 \Rightarrow p \sim 400 E_{kj} (t_{ns})^{-3} \text{ Mbar} \]

\((E_j = E \text{ in joule}; E_{kj} = E \text{ in kilojoule}; \quad 1 \text{ J/cm}^3 = 10^6 \text{ J/m}^3 = 10^6 \text{ Pa} = 10 \text{ bar} = 9.9 \text{ atm})\)
100 Mbar pressure required to drive inertial fusion targets to achieve central ignition

(see, e.g., S. Atzeni and J. Meyer-ter-Vehn, The Physics of Inertial Fusion, Oxford University Press, 2004.)
100 Mbar pressure required to implode at $u_{imp} = 350$ km/s

Assume constant pressure applied at thin hollow shell with mass $m_f$, as the radius shrinks by 50%:

$$\Rightarrow \quad \frac{1}{2} m_f u_{imp}^2 \approx <p> \left(\frac{7}{8}\right)(4\pi/3) R_0^3$$

$$<p> \approx \left(\frac{12}{7}\right) \rho_{DT} u_{imp}^2 \left(\frac{\Delta R_0}{R_0}\right) \quad (***)$$

Peak pressure $\approx 2 <p>$ (see Betti’s lecture)

$\Rightarrow$ for $R_0/\Delta R_0 = 10$, **peak pressure = 100 Mbar**

***) $\rho_{DT}$: density of solid DT = 0.25 g/cm$^3$ (need for solid $\Rightarrow$ see Betti’s lecture)
These lectures

(mostly referring to ns pulses of “moderate” intensity)

• How does laser light propagate in a plasma?
• How is laser light absorbed?
• How is ablative pressure generated?
• How is ablative pressure used to accelerate macroscopic pieces of matter?
• How efficient ablative acceleration is?
• Is ablative acceleration stable? (unfortunately not, but instabilities can be mitigated)
Laser interaction with a plasma

- Propagation

- Absorption
A free electron in the field of a plane e.m. wave, with wavenumber $k$, frequency $\omega$, average intensity $I$

$$I= \frac{c \varepsilon_0 E_0^2}{2}$$

$$\begin{align*}
F_E &= -eE \\
F_B &= -ev \times B
\end{align*}$$

$$\Rightarrow \frac{|F_B|}{|F_E|} = \frac{v}{c} \Rightarrow \text{magnetic force negligible if } v \ll c;$$

$$\Rightarrow \text{What is a typical electron velocity (in the wave field?)}$$

$$\begin{align*}
\frac{dp_y(x=0,t)}{dt} &= -eE(0,t) = -eE_0 e^{i(kx-\omega t)} = -eE_0 e^{-i\omega t} \\
\Rightarrow p_y(0,t) &= p_{osc} e^{-i\omega t}, \quad p_{osc} = \frac{eE_0}{\omega}.
\end{align*}$$

$$a_L^2 = \left( \frac{p_{osc}}{mc} \right)^2 = \frac{e^2 \lambda I}{2 \pi^2 \varepsilon_0 m^2 c^5} = 0.73 \frac{I}{10^{18} \text{Wcm}^{-2}} \left( \frac{\lambda}{1 \mu\text{m}} \right)^2 = 0.73 I_{18} \lambda_{\mu}^2$$

$$0.73 I_{18} \lambda_{\mu}^2 \ll 1 \quad \text{electrons not relativistic; magnetic force negligible}$$
(From now on non-relativistic limit)

Previously: free electron in vacuum

Now, **PLASMA**

- Electrons move to shield externally imposed fields
- Electrons collide with each other
- Electrons collide with ions

We will show that

- **Light can only propagate if** \( \omega > \omega_p \)
- **At moderate intensity** (to be defined later)
  laser energy is absorbed collisionally:

  e.m. field energy \( \Rightarrow \) electron quiver motion \( \Rightarrow \) e-i collisions
Quantitative results on propagation and absorption are provided by the dispersion relation

- For any quantity: equilibrium value + perturbation \( g = g_0 + g_1^* \)
- Linearization (neglect 2\textsuperscript{nd} order terms)
- Harmonic perturbations, \( g_1^* = g_1 \exp [i(k \cdot r - \omega t)] \)
  \[ \Rightarrow \frac{d}{dr} = i \kappa \quad \frac{d}{dt} = -i\omega \]
- Dispersion relation \( \omega = \omega(k) \) or \( k = k(\omega) \)

From the dispersion relation we obtain

- phase velocity \( v_\Phi = \omega / k \), and refraction index \( \mu = c / v_\Phi \)
- group velocity \( v_g = \frac{d\omega}{dk} \)
- absorption coefficient \( \alpha = 2\Im(k) \)

- in general, \( k \) complex, \( k = \Re(k) + i\Im(k) \)
- intensity \( I \propto E^2 \propto e^{-2\Im(k)x} e^{-2i[\Re(k)x - \omega t]} \)

exponential decay oscillating term
Deriving the dispersion relation - I

- Ions at rest
- Fluid (cold and collisional) electrons

\[
m \frac{dv}{dt} = -eE - ev \times B - mv\nu_{ei}
\]

\[J = -env\]

\[
\nabla \times E = -\frac{\partial B}{\partial t}
\]

\[
\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}
\]

Perturb above equations about the equilibrium values:
\[n = n_0; \; v_0 = E_0 = B_0 = J_0 = 0\]

\[-mi\omega v_1 = -eE_1 - mv_1\nu_{ei} \quad (1)\]

\[J_1 = -en_0v_1 = -\frac{e^2n_0}{im(\omega + iv_{ei})}E_1 \quad (2)\]

\[i\kappa(k \cdot E_1) + k^2E_1 = i\frac{\omega}{\varepsilon_0c^2}J_1 + \frac{\omega^2}{c^2}E_1 \quad (3)\]
By inserting eq. (2) into eq. (3):

\[(k^2 - \frac{\omega^2}{c^2})E_1 = -i \left( \frac{e^2 n_0}{\varepsilon_0 m} \right) \frac{\omega}{i(\omega + i \nu_{ei})c^2 E_1} \]

=> \[c^2 k^2 = \omega^2 - \frac{\omega_p^2}{1 + i \frac{\nu_{ei}}{\omega}}, \quad \text{with} \quad \omega_p = \left( \frac{e^2 n_0}{\varepsilon_0 m} \right)^{1/2}, \quad \text{plasma frequency}\]

separating real and imaginary parts

\[c^2 k^2 = \omega^2 - \frac{\omega_p^2}{1 + \frac{\nu_{ei}^2}{\omega^2}} + i \frac{\omega_p^2}{1 + \frac{\nu_{ei}^2}{\omega^2}} \frac{\nu_{ei}}{\omega} \equiv \omega^2 - \omega_p^2 + i \omega_p^2 \frac{\nu_{ei}}{\omega}\]

\[c^2 k^2 = \omega^2 - \omega_p^2 + i \omega_p^2 \frac{\nu_{ei}}{\omega}\]

\[c \frac{\mathcal{R}(k)}{2} = \sqrt{\omega^2 - \omega_p^2}\]

\[2\mathcal{S}(k) = \frac{\mathcal{S}(k^2)}{\mathcal{R}(k)} = \frac{\nu_{ei} \omega_p^2}{c \omega^2} \left(1 - \frac{\omega_p^2}{\omega^2}\right)^{-1/2}\]
Propagation (and cut-off); critical density; phase and group velocity

\[ c^2 \Re(k^2) = \omega^2 - \omega_p^2 > 0 \quad \text{for} \quad \omega > \omega_p \]

- propagation only for \( \omega > \omega_p \)
- or \( n < n_c = \frac{m\omega^2 \varepsilon_0}{e^2} = \frac{1.12 \times 10^{21}}{[\lambda(\mu m)]^2} \text{ cm}^{-3} \); critical density
- or \( \rho < \rho_c = \frac{1.86 \times 10^{-3} A}{[\lambda(\mu m)]^2 \frac{Z}{2}} \text{ g cm}^{-3} \)

- phase velocity \( v_\Phi = \frac{\omega}{k} = c \left( 1 - \frac{\omega_p^2}{\omega^2} \right)^{-1/2} > c \)

- group velocity \( v_g = \frac{d\omega}{dk} = c \left( 1 - \frac{\omega_p^2}{\omega^2} \right)^{1/2} < c \)

\[ v_g v_\Phi = c^2 \]
Refraction index always $< 1$

\[ \mu = \frac{c}{\nu \Phi} = \left( 1 - \frac{\omega_p^2}{\omega^2} \right)^{1/2} < 1 \]
Hamilton-Jacobi equations for a particle with Hamiltonian $H = H(q, p)$

$$\frac{d}{dt} q = \frac{dH}{dp}, \quad \frac{d}{dt} p = -\frac{dH}{dp}$$

For light, $q = r, p = \hbar k, \ H = \hbar \omega$, $\Rightarrow \quad \frac{d}{dt} r = \frac{d \omega}{dk}; \quad \frac{d}{dt} k = -\frac{d \omega}{dr}$

For a uniform plasma $\omega^2 = k^2 c^2 + \omega_p^2 \Rightarrow d\omega/dk = kc^2/\omega$

Actual plasmas are not uniform; nevertheless, at each point use the dispersion relation previously obtained for a uniform plasma (WKB approximation; eikonal approximation); we can then write

$$\frac{d^2 r}{dt^2} = \frac{c^2}{\omega} \frac{d}{dt} k = \frac{c^2}{\omega} \frac{d}{dr} k \frac{d}{dt} r$$

Using again the dispersion relation, and observing that $d \, r / d \, t = v_g \frac{k}{k}$

$$\frac{d^2 r}{d(ct)^2} = -\frac{1}{2} \nabla \left( \frac{\omega_p^2}{\omega^2} \right) = -\frac{1}{2} \nabla \left( \frac{n}{n_c} \right)$$
Ray tracing: a very simple, important application

Spherically symmetric plasma, cylindrical laser beam

Take a snapshot of the plasma and take time as a parameter.
Ray equation is just the equation of motion in a central potential
=> angular momentum is conserved

\[ \text{angular momentum} = \frac{h \nu}{v_g} d = \text{constant}, \text{ with } d \text{ distance from axis} \]

if \( b \) is the impact parameter of a ray at infinity (where \( v_g = c \))

\[ \implies \text{at any point } \frac{h \nu}{c} b = \frac{h \nu}{c / \mu} b \implies d = \frac{b}{\mu} \]

Off-axis rays are deflected and do not reach the critical density
Collisional (or inverse Bremsstrahlung) absorption coefficient

\[ \alpha = 2 \Im \left( \frac{\mathcal{S}(k^2)}{\Re(k)} \right) = \frac{\nu_{ei} \omega_p^2}{c \omega^2} \left( 1 - \frac{\omega_p^2}{\omega^2} \right)^{-1/2} \]

\[ \omega_p^2 \propto n_e; \quad \nu_{ei} \propto n_e T_e^{-3/2}; \quad \omega = 2\pi c / \lambda \]

=> \[ \alpha \propto n_e^2 T_e^{-3/2} \lambda^2 \left( 1 - \frac{n_e}{n_c} \right)^{-1/2} \]

Caution to the scaling with wavelength;
Absorption coefficient grows quadratically with \( \lambda \) at given density,
but accessible density decreases quadratically with \( \lambda \).

A more transparent expression is then

\[ \alpha \propto \left( \frac{n_e}{n_c} \right)^2 T_e^{-3/2} \lambda^{-2} \left( 1 - \frac{n_e}{n_c} \right)^{-1/2} \]
A more physical derivation of the absorption coefficient

\[ \nu_{\text{abs}} \xrightarrow{\text{Laser}} \text{oscillating electrons} \xrightarrow{\nu_{\text{ei}}} \text{collisional damping} \]

\[ \nu_{\text{abs}} \frac{1}{2} \varepsilon_0 E^2 = \nu_{\text{ei}} \frac{1}{2} n m v_{\text{osc}}^2 \]

\( \nu_{\text{abs}} \): absorption frequency
\( \nu_{\text{ei}} \): collision frequency

\[ \alpha = \frac{1}{L_{\text{absorption}}} = \frac{\nu_{\text{abs}}}{v_g} = \frac{\nu_{\text{abs}}}{c \mu} = \nu_{\text{ei}} \frac{n m v_{\text{osc}}^2}{c \mu \varepsilon_0 E^2} = \nu_{\text{ei}} \frac{n e^2 E^2}{c \mu \varepsilon_0 E^2 m \omega^2} \]

\[ \alpha = \left( \frac{n e^2}{\varepsilon_0 m} \right) \frac{\nu_{\text{ei}}}{c \mu \omega^2} = \frac{\omega_p^2 \nu_{\text{ei}}}{c \omega^2} \left( 1 - \frac{\omega_p^2}{\omega^2} \right)^{-1/2} \]
When does previous model apply?

- Electron must stay nearly thermal, i.e. thermalization must be faster than heating

\[ \nu_{ee} \nu_{\text{thermal}} >> \nu_{ei} \nu_{\text{osc}} \]

\[ \nu_{ee} = \nu_{ei} / Z \]

\[ \nu_{osc} = \frac{eE}{m\omega} \propto \lambda I^{1/2} \]

\[ \implies I \lambda^2 < \text{cost } TZ^{-2} \]

Inserting appropriate numerical values, for low-Z materials

\[ I (\text{W/cm}^2) \lambda^2 (\mu\text{m}) \leq 10^{14} \]

At higher intensity the absorption coefficient decreases somewhat (not too much) due to distortion of the electron distribution function (see Kruer’s book)
Absorption efficiency depends critically on $I\lambda^4$

We shall see that when a pulsed laser beam impinges on a solid target a roughly isothermal plasma “corona” is generated. In the “accessible” region, $n = n_c \exp (-x/L)$, where $x$ is the distance from the critical density and $L$ is a scalelength.

Fraction of absorbed light

$$\eta_{abs} = 1 - \exp \left( \frac{2 \int_0^\infty \alpha(x)dx}{x} \right) = 1 - \exp \left( -\frac{8\nu_{ei}L}{3c} \right)$$

$\Rightarrow$ (*)

$$\eta_{abs} = 1 - \exp \left( -\frac{I}{I^* \eta_{abs}} \right), \quad \text{where}$$

$$\frac{I^*}{10^{14} \text{Wcm}^{-2}} \approx 1.5 \frac{Z}{1 \text{mm}} \left( \frac{\lambda}{1 \mu\text{m}} \right)^{-4}$$

Absorption drops substantially when $I > I^*$

(confirmed by experiments, see next vg)

(*) Mora (1982)
Absorption efficiency improves substantially as the wavelength decreases.
Collisional and non-collisional & relativistic regimes of Laser - plasma interaction

Three regimes of interaction:

- Collisional absorption in a thermal plasma (*inverse bremsstrahlung*)
- Kinetic effects (distorted distribution function; parametric, instabilities)
- Relativistic, kinetic effects

(Approximately) characterized by the value taken by a single dimensionless parameter

\[ a_L^2 = \left( \frac{v_{\text{osc}}}{c} \right)^2 = 7.3 \times 10^{-5} I_{14} \lambda_{\mu m}^2 \]
Interaction of a laser pulse with a solid

- Ablation
- ablation pressure
a) \( t = 0 \)

plane plasma expansion

solid target

b) \( t < \frac{r_f}{u} \);

c) \( t > \frac{r_f}{u} \);

axial profiles during stage (c)

spherical expansion

transparent plasma absorbing layer

log density

\( \rho_s \)  \( \rho_0 \)  \( \rho_c \)  \( \rho_2 \)

log temperature

\( T_e \)  \( T_1 \)  \( T_2 \)

absorbing layer
“Low” intensity case: all light absorbed before reaching the critical density (Caruso and Gratton, 1968)

Ablated mass \( \frac{d(m/S)}{dt} \approx \rho_2 u_2 \)

Ablation pressure \( p \approx \rho_2 u_2^2 \)
We need to determine $\rho_2$ and $u_2$

Assume:
- Light fully absorbed before reaching critical density
- Absorbing layer self-regulates, so that $\alpha L \approx 1$
- Absorbing layer sonic (Japman-Jouget condition), $\varepsilon_{2e} \approx u^2$  

We use dimensional analysis.
First we identify the relevant dimensional quantities
- Laser intensity $I$
- Focal spot size $r_F$
- Parameter $a$ defined as follows(*):

Absorption coefficient
\[
\alpha = \frac{\omega_p^2 \nu_{ei}}{c \omega^2} \left(1 - \frac{\omega_p^2}{\omega^2}\right)^{-1/2} = a \frac{\rho_2^2}{\varepsilon_{2e}^{3/2}} \left(1 - \frac{\rho_2}{\rho_c}\right)^{-1/2}
\]

($\rho_c$ is not relevant: light does not reach it; $\varepsilon_{2e} \varepsilon_{2e} \approx u^2$)

If pulse duration long enough, expansion spherical, steady corona, time not relevant.

\[a = 2 \times 10^{29} \lambda_\mu \frac{Z^{9/2}}{A^{7/2}}\] (cgs units); dimensions $:\text{L}^8\text{T}^{-3}\text{M}^{-2}$

(*)
3 relevant quantities, 3 fundamental dimensions (L,M,T),
⇒ Unknowns $\rho_2$ and $u_2$ can be written as

$$u_2 \propto r_F^\alpha a^\beta I^\gamma \quad (1)$$

$$\rho_2 \propto r_F^\delta a^\epsilon I^\phi \quad (2)$$

with exponents uniquely determined by dimensional considerations.

For example [Eq.(1)]

$$[LT^{-1}] = [L]^\alpha [L^8 M^{-2} T^{-3}]^\beta [MT^{-3}]^\gamma$$

$$[LT^{-1}] = [L^{\alpha+8\beta} M^{-2\beta+\gamma} T^{-3\beta-3\gamma}]$$

$\Rightarrow 1 = \alpha + 8\beta$; $0 = -2\beta + \gamma$; $-1 = -3\beta - 3\gamma$

$\Rightarrow \alpha = 1/9$; $\beta = 1/9$; $\gamma = 2/9$

$$u_2 \propto r_F^{1/9} a^{1/9} I^{2/9}$$

Analogously one obtains

$$\rho_2 \propto r_F^{-1/3} a^{-1/3} I^{1/3}$$
Ablation pressure and mass ablation rate

ablation pressure \( p = \rho_2 u_2^2 \propto r_F^{-1/9} a^{-1/9} I^{7/9} \)

mass ablation rate \( -\frac{d(m/S)}{dt} = \rho_2 u_2 \propto r_F^{-2/9} a^{-2/9} I^{5/9} \)

Inserting the expression for \( a \) as function of \( \lambda \)

\[-\frac{d(m/S)}{dt} \propto r_F^{-2/9} \lambda^{-4/9} I^{5/9} \quad \quad p \propto r_F^{-1/9} \lambda^{-2/9} I^{7/9}\]

The front factors have been determined by both more detailed analysis (Mora, 1982) and numerical simulations (Gardner & Bodner, 1981), and agree with experimental values

\[-\frac{d(m/S)}{dt} \text{ [g/cm s]} = 1.6 \times 10^5 \ r_{mm}^{-2/9} \ \lambda_{\mu}^{-4/9} \ I_{14}^{5/9} f(A,Z)\]

\[
p \text{ [Mbar]} = 11 \ r_{mm}^{-1/9} \ \lambda_{\mu}^{-2/9} \ I_{14}^{7/9} g(A,Z)\]

\[
r_{mm} = r[mm]; \quad \lambda_{\mu} = \lambda[\mu m]; \quad I_{14} = I[1014 \ W/cm^2]; \quad f = g = 1 \text{ for plastic}\]
Previous expression for ablation pressure agrees with large set of experimental data collected between 1965 and 1995

For details and reference to original data: SA, PPCF 1987, 2000; SA & MtV’s book
When do previous model apply?

Corona of size $r \geq r_F$ must form:

$\Rightarrow$ Pulse duration $> r_0/u_2 \approx 1.5 \; r_{\text{mm}}^{8/9} \; \lambda_{\mu}^{-2/9} \; I_{14}^{-2/9} \; g(A,Z)$ [ns]

Light is absorbed at density $\rho_2 << \rho_c$

$\Rightarrow \quad I_{14} << 4 \; r_{\text{mm}} \; \lambda_{\mu}^{-4} \; I_{14}^{-2/9}$ [ns]

Note the very strong wavelength scaling
“High” intensity \((10^{14} - 10^{15} \text{ W/cm}^2, @ \lambda = 0.35 - 0.5 \text{ µm})\):
light absorbed at the critical density
(see, eg., Manheimer et al., *Phys Fluids* 1982; Lindl, *Phys. Plasmas* 1995)

Assume
- light absorbed at critical density
- critical density = sonic point (sound velocity: \(c_s\))
- corona expansion: isothermal rarefaction wave

\[ E/S = 4 \rho_c c_s^3 t \quad \text{(see SAMtV book, Ch. 7)} \]
\[ I = (E/\text{St}) = 4 \rho_c c_s^3 \quad \Rightarrow \quad c_s = (I/4\rho_c)^{1/3} \]
\[ p \approx 2 \rho_c c_s^2 \propto (I/\lambda)^{2/3} \quad \text{d(m/S)/dt} \approx \rho_c c_s \approx (I/\lambda^4)^{1/3} \]

With front factor (see Lindl 1995, or Betti’s lecture):

\[
p = 8.7 \left( \frac{I_{14}}{\lambda_{\mu}} \right)^{2/3} \text{ Mbar}
\]

Cfr with recent simulations in spherical geometry (SA et al, NJP 2013)
\[ p = \text{cost } I^{0.75} \lambda^{-0.40} \quad \text{d(m/S)/dt} = \text{cost } I^{0.6} \lambda^{-0.40} R^{-0.22} \]
Ablative acceleration of plane and spherical targets

Rocket model of ablation
Irradiation of a “tick” disk, plasma formation
Ablative acceleration of a “thin” disk
Laser acceleration = acceleration of a rocket (of mass \( m(t) \))

No exerternal forces: momentum is conserved

\[
d \mathbf{Q}_{\text{ex}} + d \mathbf{Q}_{\text{rocket}} = 0
\]

\[
d(mv) + [-dm(u_{\text{ex}} + v)] = 0
\]

\[=> \quad dv = u_{\text{ex}} \frac{dm}{m}
\]

\[
v(t) = -u_{\text{ex}} \ln \frac{m(t)}{m_0}
\]
rocket efficiency (in laser fusion *hydrodynamic efficiency*)

\[
\eta_{h-ideal} = \frac{E_{\text{payload}}}{E_{\text{payload}} + E_{\text{exhaust}}} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}(m_0 - m)u_{ex}^2} = \frac{x \ln x^2}{1 - x}
\]

For laser drive, after some algebra:

\[
\eta_h = \frac{1}{2} \frac{mv^2}{E_{\text{laser-absorbed}}} = \frac{1}{2} \frac{d(m/S)}{dt} \frac{u_{ex}^2}{I_{\text{abs}}} x \ln x^2 = h \frac{x \ln x^2}{1 - x}, \quad h \approx 0.25 \text{ for low-Z materials}
\]
Hollow shell target, irradiated by a large number of overlapping beams

Target (hollow shell)
- Fuel mass: few mg
- Radius: 1 – 3 mm
- Fuel radius / thickness = 10

Laser driver pulse
- Energy: 1 – 5 MJ
- Duration: 10 – 20 ns
- Peak power: 300 – 500 TW
- Peak intensity: $10^{15}$ W/cm²
- Wavelength: $(1/4) – (1/3) \mu m$

Compressed fuel
- Density: 200 – 1000 g/cm³
- Low average entropy, but hot-spot with $T = 10$ keV

\[
\begin{align*}
R_a &= 1.971 \text{ mm} \\
R_0 &= 1.934 \text{ mm} \\
R_i &= 1.760 \text{ mm}
\end{align*}
\]
Irradiation, implosion, compression, ignition & burn
(shell with 1.67 mg of DT fuel, irradiated by 1.6 MJ pulse, see later)

3 mm density (g/cm$^3$) 10$^{-4}$ 10$^{-1}$ 10$^{2}$ temperature (K) 10$^{2}$ 10$^{5}$ 10$^{8}$

3 mm simulated interval = 25 ns

S. Atzeni, 1992
Zoom (in space and time): final compression, ignition, burn and explosion

simulated time $= 0.5 \text{ ns}$
Laser absorption $\Rightarrow$ ablation

high temperature plasma

very high pressure

corona expands,

shocks launched in the shell
Shocks driven by laser absorption

implies shell inward acceleration (implosion) = rocket
The ablative pressure drives the implosion of the shell.

At stagnation substantial fuel compression and heating occur (see curves 5).

1) $t = 0$
2) $t = 20$ ns
3) $t = 22.4$ ns
4) $t = 23.6$ ns
5) $t = 24.46$ ns
At the end of the implosion a **hot spot** is formed, which suddenly self-heats and drives a fusion burn wave (notice the time scales)
(In)stability of
Laser-accelerated targets
Rayleigh-Taylor instability
unavoidable in inertial fusion

deceleration-phase instability at the hot spot boundary
(2D simulation)

Atzeni & Schiavi, PPCF 2004
Rayleigh instability of interface in hydrostatic equilibrium

Taylor instability of accelerated interface; equivalent to Rayleigh instability if analysed in a frame moving with the interface
Rayleigh, 1883

Taylor, 1950

\( \vec{g} = -\vec{a} \)

\( \vec{a} \uparrow \quad \vec{v} \uparrow \)

\( \rho_z \)

\( \rho_1 \ll \rho_z \)

\( \rho_1 > \rho_z \)

\( t = 0 \)

Power

(Laser, explosive, ...)

(*) To write Newton's 2nd law in the slab's frame.
Rayleigh instability, single mode and multimode, nonlinear evolution

$t_0$ $t_1 > t_0$ $t_2 > t_1$

Bubble Spike
Fig. 3. Evolution of the multi-wavelength Cartesian Rayleigh-Taylor instability. The initial seed is made up by the superposition of (cosine) modes $m = 5$ and $m = 8$ ($\lambda_5 = \lambda_1/5$ and $\lambda_8 = \lambda_1/8$). Density ratio at the interface $\rho_1/\rho_2 = 10$. Mesh: 64x400.
Classical Rayleigh instability of superposed fluids

Linear theory:

Sinusoidal perturbations of the interface between incompressible, ideal fluids, with wavelength $\lambda$ (i.e. wave number $k = 2 \pi/\lambda$) and “small” initial amplitude $\xi_0$, grow exponentially in time

$$\xi = \xi_0 \exp(\gamma t) \quad [\text{for } \xi \leq \lambda/10]$$

with growth rate

$$\gamma = \gamma_{cl} = \sqrt{Agk}$$

$$A = \sqrt{\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}}, \quad \text{Atwood number}$$
Rayleigh instability of superposed fluids
growth rate derivation from a simple energy principle

\[ \delta U = -q \left< \int_0^\xi dy \left( p_2 - p_1 \right) y \right> = \left( p_2 - p_1 \right) \frac{q}{2} \left< \xi^2 \right> \quad (1) \]

**Kinetic energy variation:**
\[ \delta T = \left< \int_{-\infty}^{\infty} dy \frac{1}{2} \rho \left( v_x^2 + v_y^2 \right) \right> \quad (2) \]

1. linearity \( \rightarrow \) sinusoidal, small perturbations
2. incompressibility \( \rightarrow \) \( \nabla \cdot \mathbf{u} = 0 \)
3. ansatz for \( y \) dependence: \( e^{-kl y} \)
\[ v_x = \frac{|y|}{y} A(t) e^{-k|y| \sin kx} \]
\[ v_y = A(t) e^{-k|y| \cos kx} \]
\[ S(0) = v_y \]

(2) \[ ST = \frac{1}{2k} (p_1 + p_2) <s^2> \quad (3) \]

Now we look for solutions with exponential time behaviour
\[ \xi = e^{yt} \xi_0 \quad [\text{or } \dot{\xi} = \delta_\xi] \quad (4) \]

and impose energy conservation
\[ ST + S U = 0 \quad (5) \]

Inserting (4) in (3) and (2), and the eqs. Thus obtained into (5) we get:
\[ <\dot{\xi}^2> = <\xi^2> \frac{p_2 - p_1}{p_1 + p_2} k \]
\[ A = \text{Atwood number} \]
or \( \delta^2 = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} g k = A g k \) (6)

- \( \rho_2 > \rho_1 \) instability
- \( \rho_2 < \rho_1 \) undamped oscillations

\[ \gamma = \gamma_{cl} = \sqrt{A g k} \]

\[ A = \frac{\sqrt{\rho_2 - \rho_1}}{\sqrt{\rho_2 + \rho_1}}, \quad \text{Atwood number} \]

- RTI of a layer of finite thickness: G. I. Taylor (1950)
If RTI growth at ablation front were classical, ICF would not be feasible

Linear growth a perturbation of initial amplitude $\xi_0$ and wavelength $\lambda$:

$$\xi(\lambda, t) = \xi_0 \exp\left( \int_0^t \gamma(\lambda, t') \, dt' \right) = \xi_0 \exp[\Gamma(\lambda, t)]$$

Assume

- $\gamma = \sqrt{Aak} \equiv \sqrt{2\pi a / \lambda}$ ( $A \equiv 1$ at ablation front)
- constant acceleration between $r = R_0$ and $r = (1/2) R_0$
  
  $= > at^2 = R_0$
- worst (most dangerous) mode for a layer of thickness $\Delta R$:
  $\lambda = \pi \Delta R$ (for shorter wavelengths linear growth saturation)

$= =>$ for the worst mode, at the end of acceleration stage

$$\Gamma \approx \sqrt{\frac{2R_0}{\Delta R}} = \sqrt{\frac{2R_0 \, R}{R \, \Delta R}} > 11, \quad \text{i.e.} \quad \xi > 60 \, 000 \, \xi_0 \quad \text{clearly unacceptable}$$

(here we have assumed $R = R_0/2$, and $R/\Delta R > 30$)
RTI, classical vs ICF
ablation reduces growth, stabilises “short” wavelengths

classical RTI: uniform density, sharp interface, no fluxes, incompressible fluids

in ICF, ablative RTI: continuous density; mass, momentum & energy fluxes (ablation), [+ compressible fluids, + spherical geometry + ...]

ablative RTI:
growth rate reduced, short wavelengths stable

Theory vs experiment
(Budil et al. 1996)

theory vs simulation
(Tabak et al. 1990)
The major differences with classical RTI are caused by (*):
- finite density gradient (with minimum density scale-length $L_{\text{min}}$)
- ablation (with surface rate $d\mu/dt$)

Theory (Sanz 1994, Betti et al 1996, 1998) is involved. However results are fitted well by (Betti et al. 1998, generalising Takabe et al. 1985)

$$\gamma = \alpha_{\text{RT}} \sqrt{\frac{ak}{1 + kL_{\text{min}}}} - \beta_{\text{RT}} ku_{a}$$

where $\alpha_{\text{RT}} = 0.9 - 1$ and $\beta_{\text{RT}} = 1 - 3$ are coefficients depending on laser intensity, light wavelength and material,

$$u_{a} = \frac{dm/dt}{\rho_{pa}}$$ is the ablation velocity, with $\rho_{pa}$ peak density ahead of the ablation front

(*) For detailed treatment and bibliography see Ch. 8 of Atzeni & Meyer-ter-Vehn, 2004.
Ablative stabilisation: theory, simulations, experiments

Theory vs experiment (Budil et al. 1996)

Takabe-like formula vs simulation (Tabak et al. 1990)
Ablative RTI allows for significant reduction of the growth factor $\Gamma$ from ablative RTI linear growth rate and rocket implosion model (Lindl et al 2004)

Indirect drive:
high implosion velocity: strong stabilisation, $\Gamma$ acceptable
Ablative RTI allows for significant reduction of the growth factor $\Gamma$

from ablative RTI linear growth rate and rocket implosion model (Lindl et al 2004)

for direct drive:

$$\Gamma_{\text{max}} = \frac{8.5}{\alpha_{if}^{2/5} I_{15}^{1/15}} \left( \frac{u_{\text{imp}}}{3 \times 10^7 \text{cm/s}} \right)^{1.4}$$

reduction with respect to classical case, but still insufficient, unless in-flight isentrope factor $\alpha_{if}$ is raised to 3 – 5

Solution: increase isentrope factor at the outer surface of the shell, without affecting inner layers: ADIABAT SHAPING (several different scheme, by all major labs. They work!)
Example of adiabat shaping (for a shock ignition target):
large RTI growth reduction

Atzeni, Schiavi, Bellei 2007, confirmed by Olazabal et al (2011), and Atzeni et al (2011)

adiabat shaping RX2 technique
(Anderson & Betti, 2004)
Ablative stabilisation also affects deceleration-phase Rayleigh-Taylor instability

- In ICF, dense shell ablated by heat flux and fusion $\alpha$-particles from the hot spot (Guskov and Rozanov 1976, Atzeni and Caruso 1981; 1984)

- Lobatchev and Betti [PRL, 85 (2000)]:
  ablation stabilizes dp-RTI, just in the same way as at a radiation/laser-driven front
Experiment on single mode RTI of radiation driven foil
(Lindl et al, 2004)

FIG. 6-6. (Color) Experiments on plasma targets have allowed quantitative evaluation of the growth of RT instability in the presence of radiation dilution. (a) Face-on streaked images provide a 1D history of the time variation in the spatial distribution of the x-ray backlighting transmission through the perturbed sample. (b) Side-on gated images provide a 2D record of the spatial distribution of the perturbed foil. (c) Side-on streaked images provide a record of the foil position as a function of time.
RTI in ICF: instability at ablation front & instability at stagnation (Lindl et al, 2004) perturbation fed from outer to inner surface of the shell
Simulation by the NRL group; Bates et al., HEDP 2010; Schmitt et al., PoP 2010
Rayleigh-Taylor instability hinders hot spot formation and ignition
(multimode perturbation with rms amplitude at the end of the coasting stage = 1.5 \(\mu m\))

Ion temperature (eV) map evolution

S. Atzeni and A. Schiavi, 2004
Rayleigh-Taylor instability hinders hot spot formation and ignition (multimode perturbation with rms amplitude at the end of the coasting stage = 1.5 µm)

Ion temperature (eV) map evolution

S. Atzeni and A. Schiavi, 2004
Inner surface RTI in ICF

(Atzeni & Schiavi, PPCF 2004)
RTI limits the size of the hot spot

Below: density maps at the same time (290 ps) for cases with different perturbation amplitude:
The size of the hot spot [see the 10 keV (red) and 5 keV (orange) contours] is reduced by the penetration of the RTI spikes.

(Atzeni & Schiavi, PPCF 2004)
The end